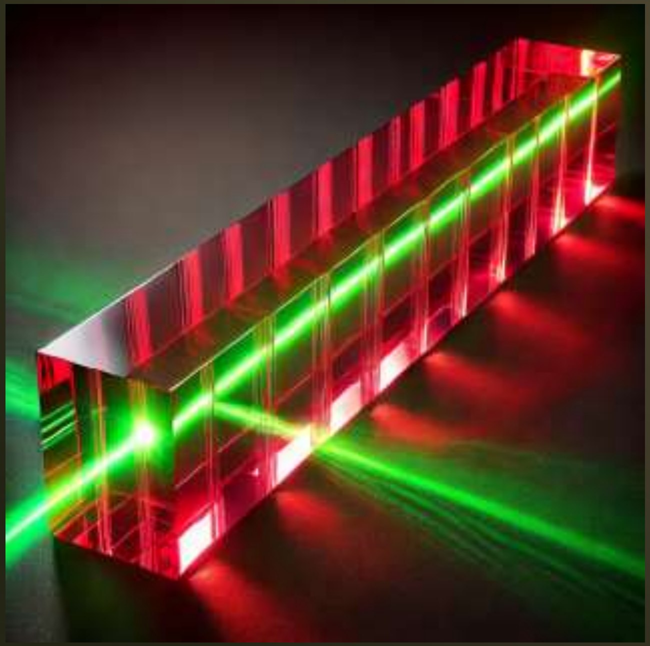
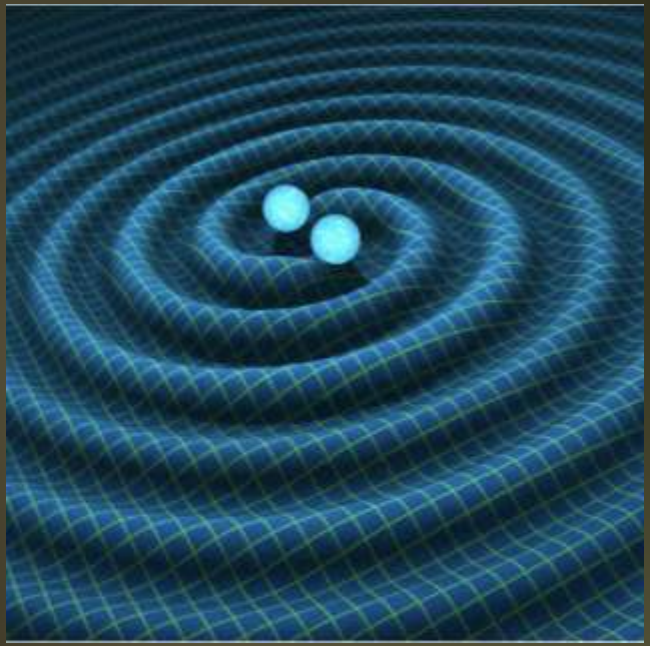




Squeezed States of Light for GW Detection

PHYSICS CONCERTO 2024

TIMOTHY ARAUJO

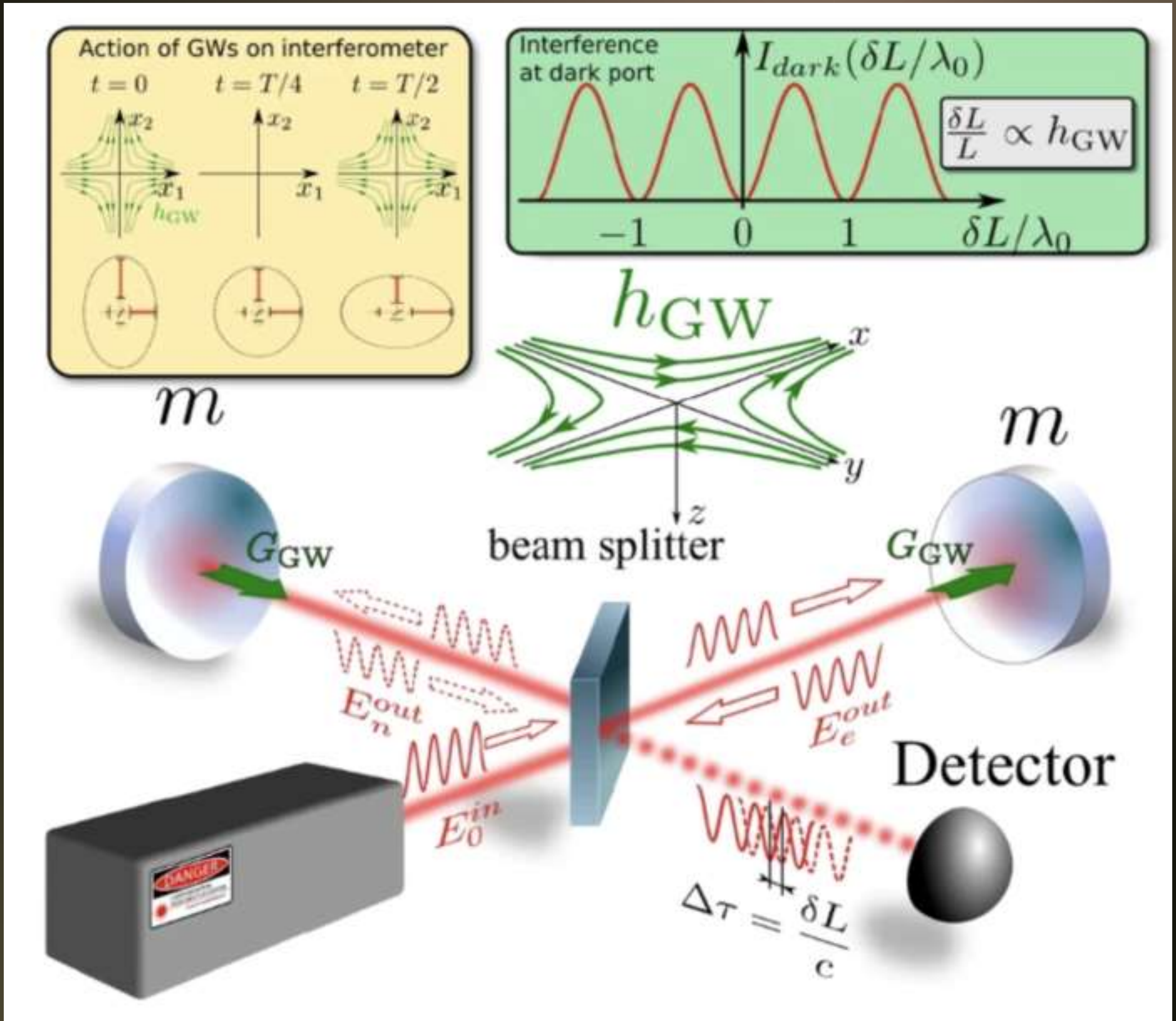
Outline

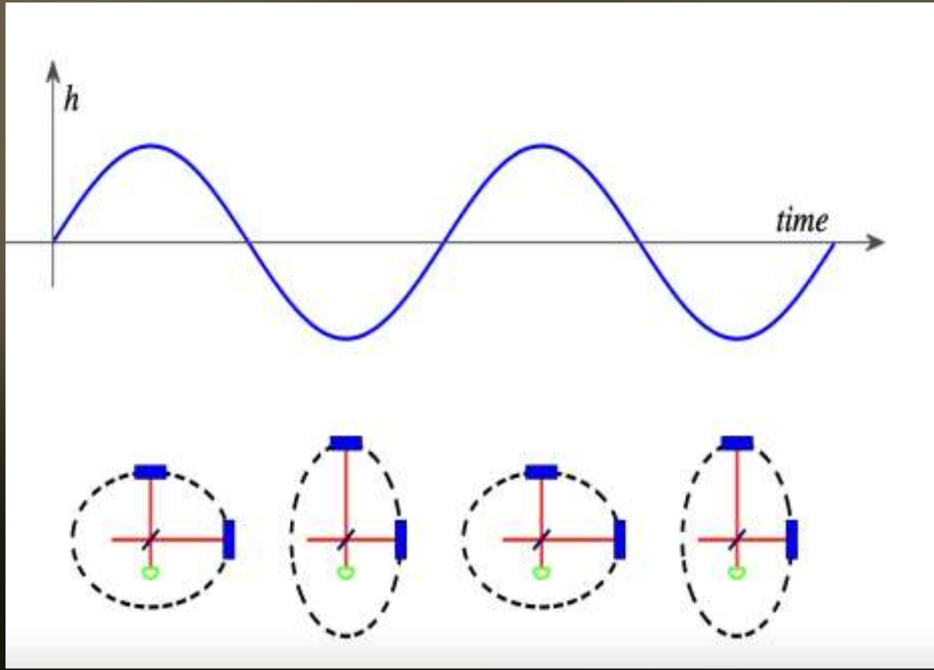
- Gravitational Waves
- How are GW's Detected
  - Interferometer
  - Initial LIGO
- Interferometer Noise
  - Advanced LIGO Noise
- Field Quantization
  - Quantum Harmonic Oscillator
  - Zero Point Energy
- Quadrature Operators
- Coherent States
  - Phase Space Representation
- Squeezing
  - Classical Squeezing
  - Quantum Squeezing: Degenerate Optical Parametric Process
  - Squeezing Light Implementation

Gravitational Waves

- Caused by catastrophic cosmic events
- 1st detection (GW150914) on Sep 2015 by NSF LIGO
  - Michelson Interferometer
- Gravitational Radiation is weak
  - Large Distances of propagation
  - Gravity weakest of 4 forces in Nature
- Space-Time disturbance 10,000x smaller than nucleus!
- Detection Offers:
  - Test for General Relativity in Strong Gravity regime
  - Astrophysical Origin
    - Source Mass and Distances
  - Dynamics, EOS high density nuclear matter, etc
  - Window into the early Universe!

Interferometer



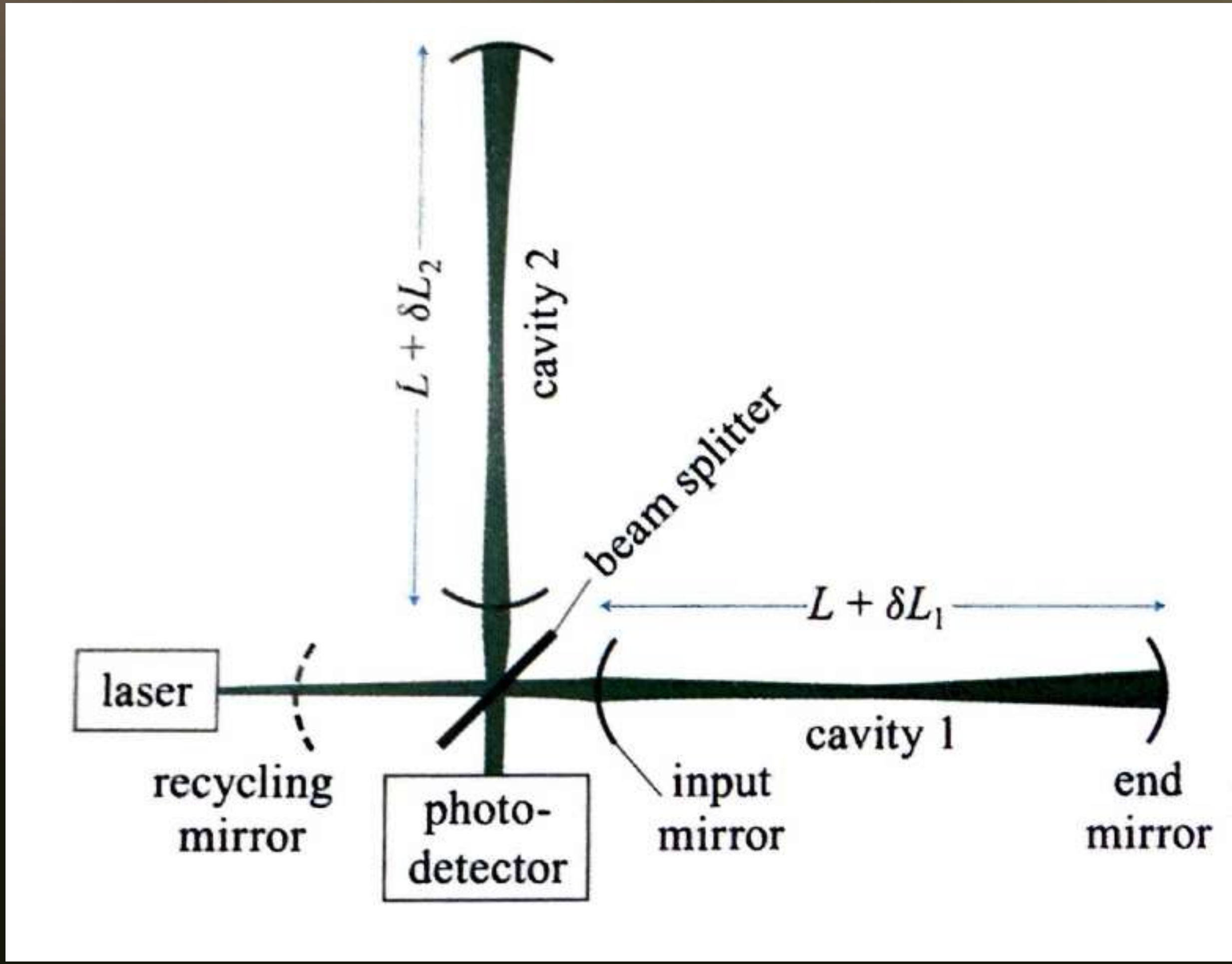


Danilishin, S.L., Khalili, F.Y. & Miao, H. Advanced quantum techniques for future gravitational-wave detectors. *Living Rev Relativ* **22**, 2 (2019). https://doi.org/10.1007/s41114-019-0018-y

Michelson Interferometer (Initial LIGO)

- Made 1st direct observation of GW's by BBH merger (36 $M_\odot$ & 29 $M_\odot$)
- Inclusion of Fabry-Perot sub-cavities offer compactness and sensitivity
  - Insensitive to fluctuations in frequency
- Recycling mirror cavity
  - Recycles Light -> 50x more Power Circulation
- Without other noises can reach shot noise limited sensitivities ~ $1/\sqrt{N}$
  - 50x increase in power -> $\Delta h_{new} \sim \frac{\Delta h_{old}}{\sqrt{50}} \sim \frac{1}{7} \Delta h_{old}$
  - Quantum limits anything better
- Classical Noises
  - Thermal noises, scattering, deformations, acoustic noise, seismic & gravity gradient fluctuations, etc

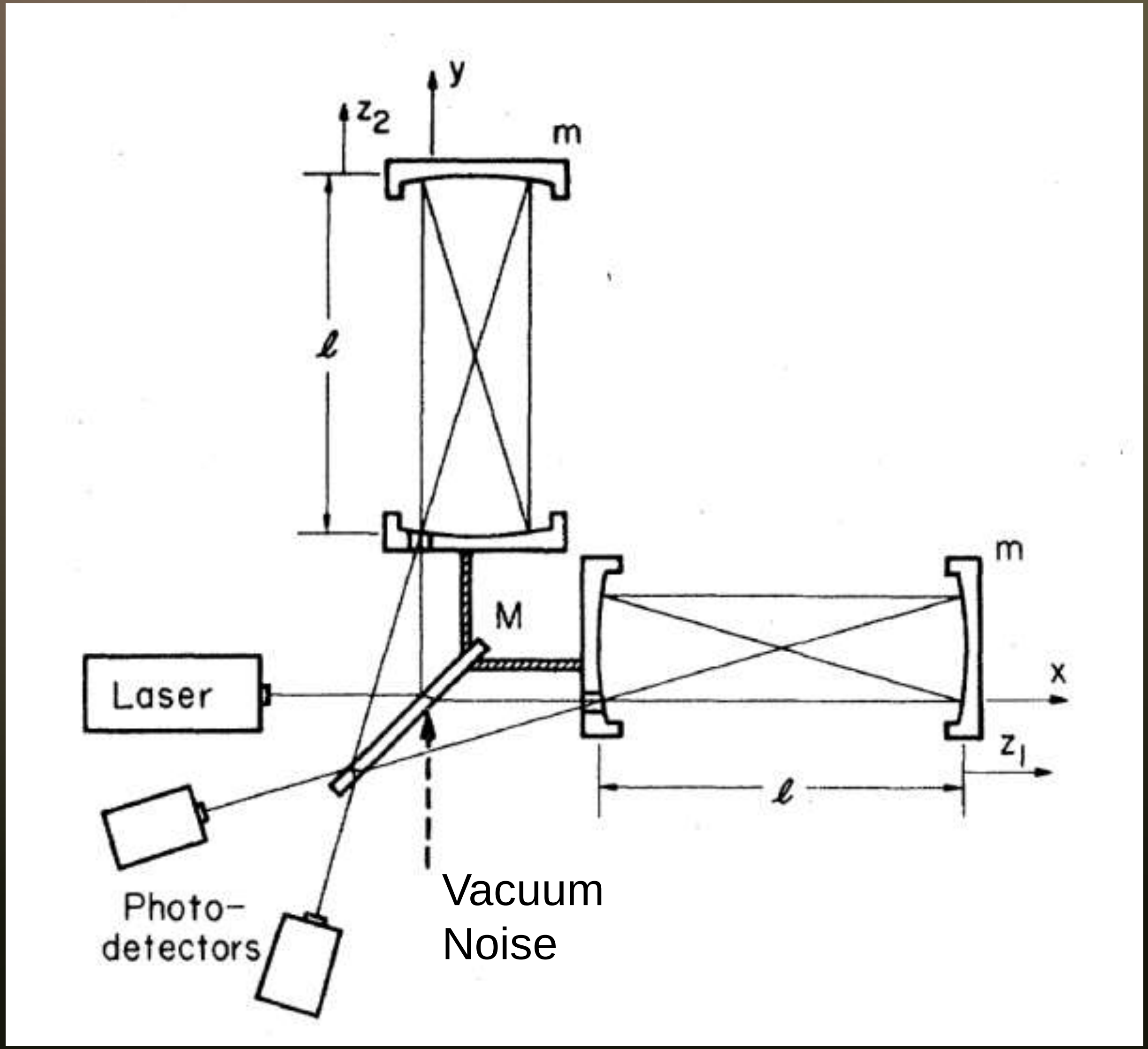
1st generation LIGO Detector



Quantum Noises

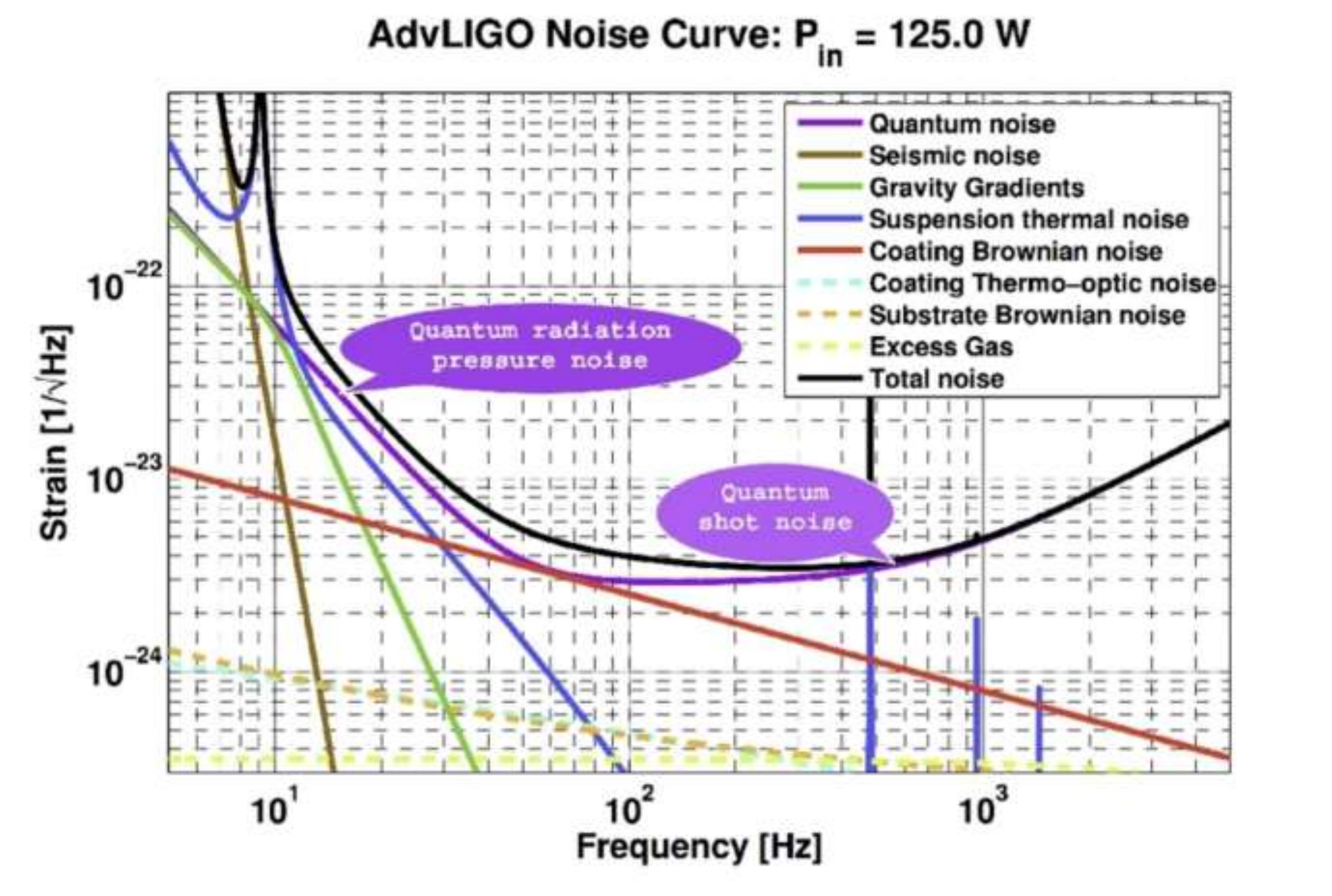
- Quantum Noise – Quantum Fluctuations in Phase and Amplitude of Light
  - Quantum Shot Noise (Uncertainty in Arrival time of Photon $\hat{x}$) - Noise in Phase
  - Quantum Radiation Pressure Noise (Uncertainty in Amplitude $\hat{p}$) – Creates differential beating radiation pressure on mirrors (Avg Radiation Pressure = $\frac{I}{c} = \frac{1}{2}\varepsilon_0 |E|^2$)
- Cannot have simultaneously vanishing uncertainties for a single mode $[\hat{x}, \hat{p}] = i\hbar$
- QM noises attributed to vacuum fluctuations caused by zero-point energy
- Enters through un-used interferometer input port

Vacuum Noise



Advanced LIGO Noise

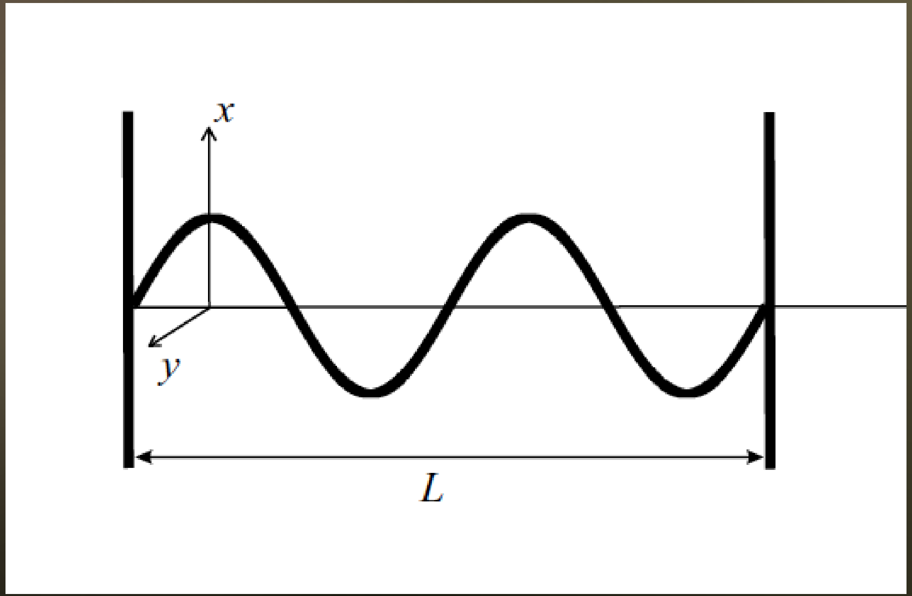
- Different noises dominate at different GW frequencies (particularly QM noise)
- Quantum Radiation Pressure limits strain sensitivity at low frequency
  - Low Frequency important for:
    - Detecting Massive BBH Merger
    - Better Parameter Estimation
    - Allow for localization of EM counterpart
- Shot Noise limits sensitivity higher frequency bands
  - detection of spectrum of merger and ringdown phase of BNS system
  - Can reveal physics of dense nuclear matter



Quantized EM Field

Maxwells Equations

$$\nabla \cdot E = 0$$
$$\nabla \cdot B = 0$$
$$\nabla \times E = \frac{\partial B}{\partial t}$$
$$\nabla \times B = \mu_o \epsilon_o \frac{\partial E}{\partial t}$$



Solutions:

$$\hat{E}_x(z,t) = \frac{2\omega^2}{V\epsilon_0} q(t)\sin(kz)$$
$$\hat{B}_y(z,t) = \left(\frac{\mu_0\epsilon_0}{k}\right)\left(\frac{2\omega^2}{V\epsilon_0}\right)\dot{q}(t)\cos(kz)$$
$$\hat{H} = \frac{1}{2}\int \epsilon_0 E_x^2 + \frac{1}{\mu_0} B_y^2\, dV = \frac{1}{2}(p^2 + \omega^2 q^2)$$

Harmonic Oscillator!

Harmonic Oscillator

- p, q can be replaced by operators
  - Correspondence rule
- Hamiltonian becomes operator and expressed in terms of a & $a^\dagger$

$$\hat{H} = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right), \qquad \hat{n} = \hat{a}^\dagger\hat{a}$$

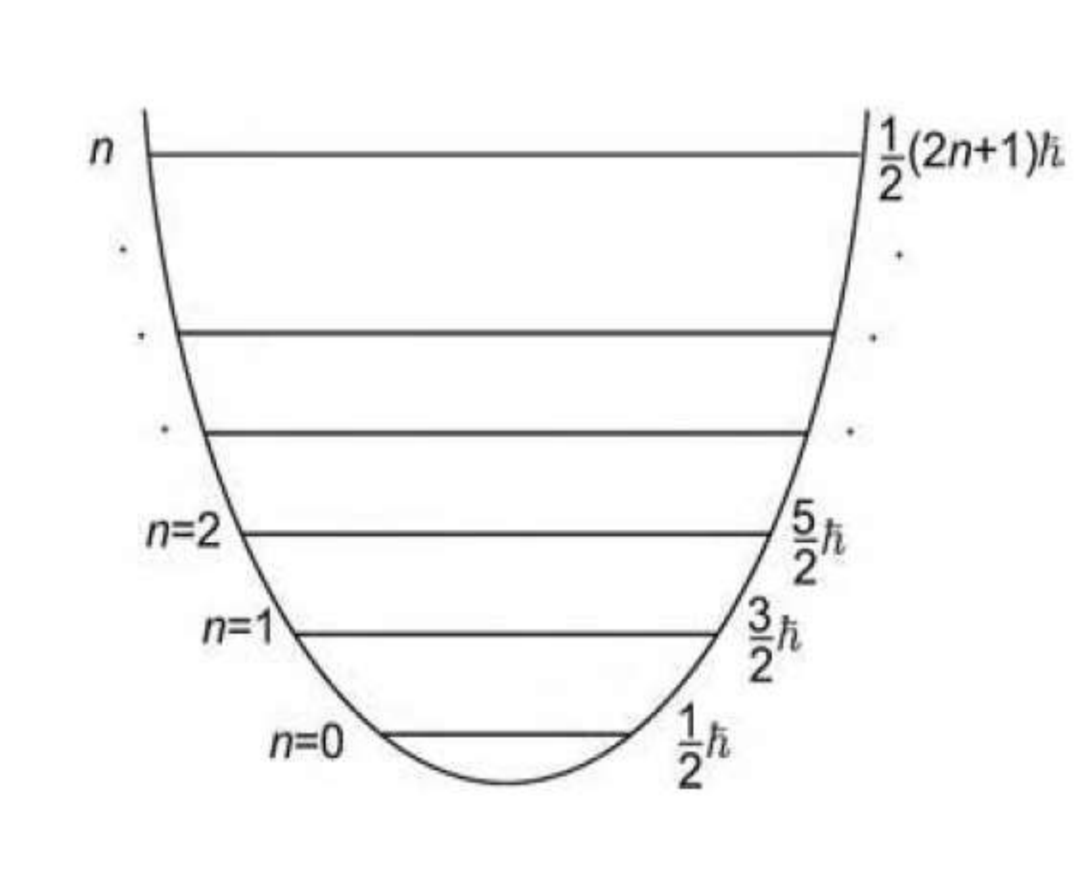
- $|n\rangle$ is energy eigenstate

$$\hat{H}|n\rangle = E_n|n\rangle$$

Where $E_n = \hbar\omega(n + {}^1/_2)$

- For vacuum state $|0\rangle$

$$\hat{H}|0\rangle = \frac{1}{2}\hbar\omega|0\rangle \text{ (Zero Point Energy)}$$



Quadrature Operators

- Introducing non- Hermitian raising (creation) and lowering (annihilation) operators $\hat{a}^{\dagger}$, $\hat{a}$

$$\hat{a} = (2\hbar\omega)^{-\frac{1}{2}}(\omega\hat{q} + i\hat{p})$$

$$\hat{a}^{\dagger} = (2\hbar\omega)^{-\frac{1}{2}}(\omega\hat{q} - i\hat{p})$$

$\hat{E}_x$ becomes:

$$\hat{E}_x(z,t) = \left(\frac{\hbar\omega}{\varepsilon_0 V}\right)^{\frac{1}{2}}\left(\hat{a} + \hat{a}^{\dagger}\right)\sin(kz)\,,$$

Given $\hat{X}_1 = \frac{1}{2}\left(\hat{a} + \hat{a}^{\dagger}\right),\ \ \hat{X}_2 = \frac{1}{2i}(\hat{a} - \hat{a}^{\dagger}),$

$$\hat{E}_x(z,t) = 2\left(\frac{\hbar\omega}{\varepsilon_0 V}\right)^{\frac{1}{2}}\sin(\mathrm{kz})[\hat{X}_1\cos(\omega t) + \hat{X}_2\sin(\omega t)]$$

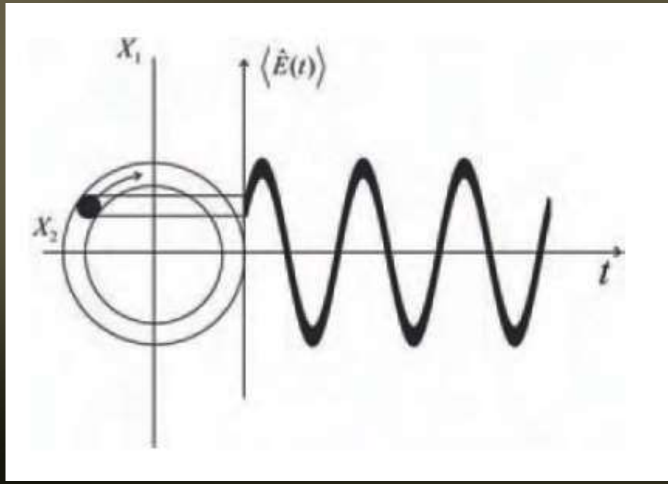
Quantum Noise is described by variances in the observables $\hat{X}_1$, $\hat{X}_2$ imposed by quantization of Light!

Coherent States (Classical Quantum States)

- We seek a Quantum State that exhibits classical properties in the classical limit
- Classical Quantum States are coherent states $|\alpha\rangle$ (eigenstates of non-Hermitian annihilation operator ie $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$ where $\alpha = |\alpha|e^{i\theta}$)
  - Found by unitary displacement operator of vacuum state $|\alpha\rangle = D(\alpha)|0\rangle = e^{-\frac{|\alpha|^2}{2}} e^{\alpha a^{\dagger}}|0\rangle$
  - Defining complex amplitude operator $\hat{\alpha} = \hat{X}_1 + i\hat{X}_2$
  - $\langle \hat{X}_1 + i\hat{X}_2 \rangle = \alpha$
  - Uncertainties in observable quadratures are symmetric
    - $\Delta X_1 = \Delta X_2 = \frac{1}{2}$
  - Mean photon # $\langle N \rangle = \langle \alpha | a^{\dagger} a | \alpha \rangle = |\alpha|^2$
  - Variance in photon # $\Delta N = |\alpha|$

Coherent States (Classical Quantum States)

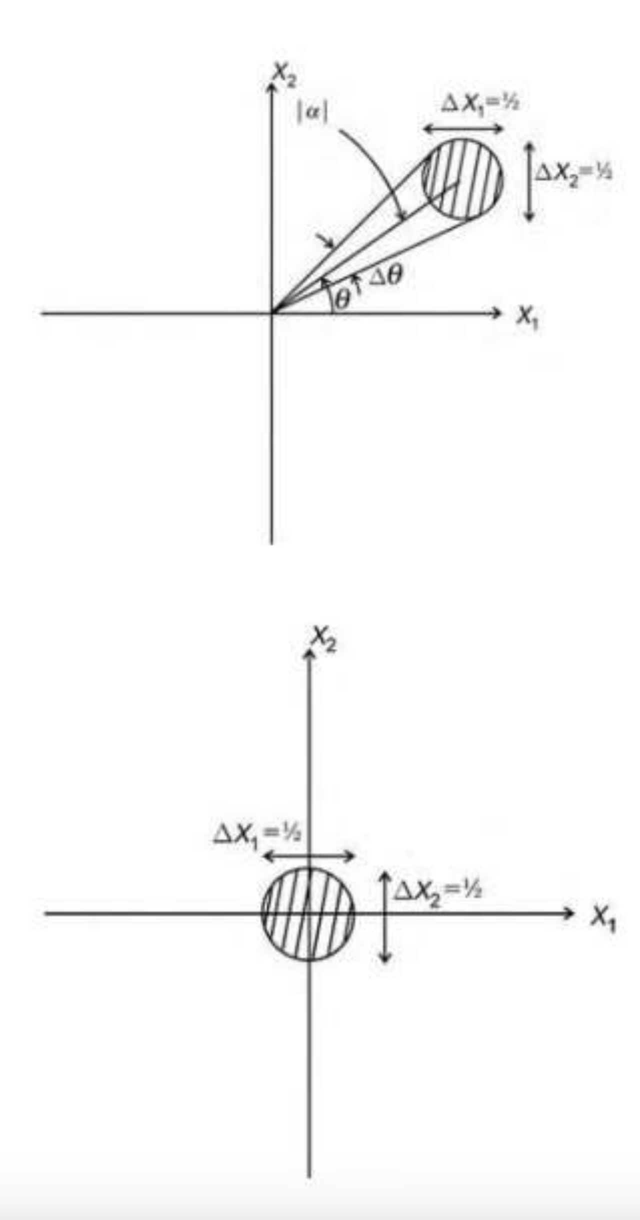
- Mean field $\langle \alpha | \hat{E}_x | \alpha \rangle = 2|\alpha| \left(\frac{\hbar\omega}{2\epsilon_0 V}\right)^{\frac{1}{2}} \sin(\omega t - k \cdot r - \theta)$
- With fluctuations $\langle \alpha | \hat{E}_x^2 | \alpha \rangle = \frac{\hbar\omega}{2\epsilon_0 V} [1 + 4|\alpha|^2 sin^2(\omega t - k \cdot r - \theta)]$
- Std dev = $\sqrt{\langle \hat{E}_x^2 \rangle} = \left(\frac{\hbar\omega}{2\epsilon_0 V}\right)^{\frac{1}{2}}$



Coherent State
Phase Space Portrait

Coherent State

Vacuum State



Classical Squeezing (Driven Harmonic Oscillator)

- Suppose we modulate HO potential with parametric drive at 2x the natural frequency

$$V(x) = 1/2m\omega_0^2 x^2(1 + \varepsilon \sin(2\omega_0 t))$$

EOM: $\ddot{x} = -\omega_0^2 x(1 + \varepsilon \sin(2\omega_0 t))$

Ansatz: $x(t) = C(t)\cos(2\omega_0 t) + S(t)\sin(2\omega_0 t)$

Assume C and S vary slowly such that $\ddot{C} = \ddot{S} = 0$

$$-\omega_0 \dot{C}\sin(\omega_0 t) + \omega_0 \dot{S}\cos(\omega_0 t) = \omega_0^2 \varepsilon \sin(2\omega_0 t)(C(t)\cos(\omega_0 t) + S(t) sin(\omega_0 t))$$

$$-\omega_0 \dot{C}\sin(\omega_0 t) + \omega_0 \dot{S}\cos(\omega_0 t) = -\frac{\omega_0^2 \varepsilon}{2}[C(t)\sin(\omega_0 t) + S(t)\cos(\omega_0 t) + 3\omega_0\ terms]$$

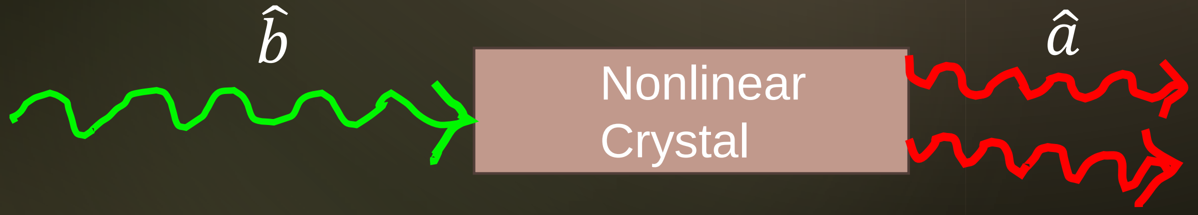
Coeff Diff Eqs: $\dot{C} = -\frac{\omega_0^2 \varepsilon}{2}\mathrm{C}$, $\dot{S} = \frac{\omega_0^2 \varepsilon}{2}\mathrm{S}$

$$C(t) = C(0)e^{-\frac{\omega_0^2 \varepsilon}{2}t},\ S(t) = S(0)e^{\frac{\omega_0^2 \varepsilon}{2}t}$$

Quantum Squeezing

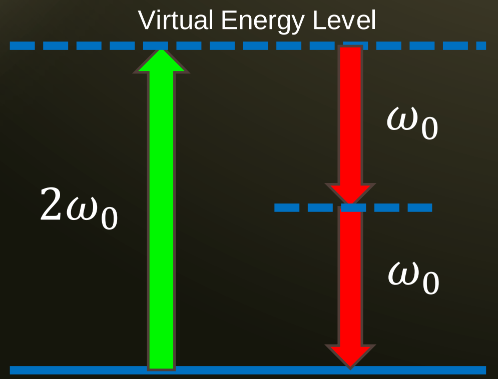
- Need a parametric drive at $2\omega_0$
- Quantum harmonic oscillator is mode of EM field
- Need to couple harmonic oscillator at $2\omega_0$ with oscillator at ω_0
- Need nonlinear physics to allow interactions between photons

- Nonlinear Physics requires a strong laser pump at $2\omega_0$ and nonlinear crystal (e.g KDP)



$$\hat{H} = \hat{a}\hat{a}\hat{b}^{+} + \hat{a}^{+}\hat{a}^{+}\hat{b}$$

Energy Level Diagram



Degenerate Parametric Down Conversion

Cont'd

- Let $\beta = \frac{r}{2}e^{i\varphi}$ be a strong coherent state ($|\beta| \gg 1$)

$$\hat{H}|\beta\rangle = \left(\hat{a}\hat{a}\hat{b}^{\dagger} + \hat{a}^{\dagger}\hat{a}^{\dagger}\hat{b}\right)|\beta\rangle = \hat{a}\hat{a}\hat{b}^{\dagger}|\beta\rangle + \hat{a}^{\dagger}\hat{a}^{\dagger}\hat{b}|\beta\rangle$$

For ($|\beta| \gg 1$): $\hat{b}^{\dagger}|\beta\rangle \approx \beta^{*}|\beta\rangle$, $\hat{b}|\beta\rangle = \beta|\beta\rangle$

$$\hat{H} = \frac{r}{2}[\hat{a}^{2}e^{-i\varphi} + (\hat{a}^{\dagger})^{2}e^{i\varphi}]$$

Time-evolution of coherent state given by $e^{-i\hat{H}t}$

For fixed time t

Squeeze operator: $S(r) = e^{-\frac{r}{2}(e^{-i\varphi}\hat{a}^{2} - e^{i\varphi}(\hat{a}^{\dagger})^{2})}$ and is unitary

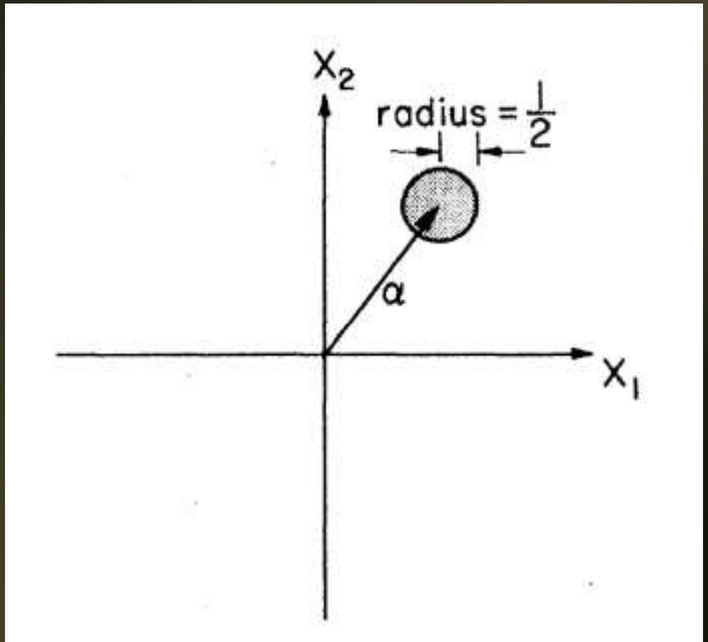
Cont’d

In Heisenberg picture, operators transform operators

$S(r)(\hat{Y}_1+i\hat{Y}_2)S^{\dagger}(r) = \hat{Y}_1 e^{-r} + i\hat{Y}_2 e^{r}$ where r is squeezing factor

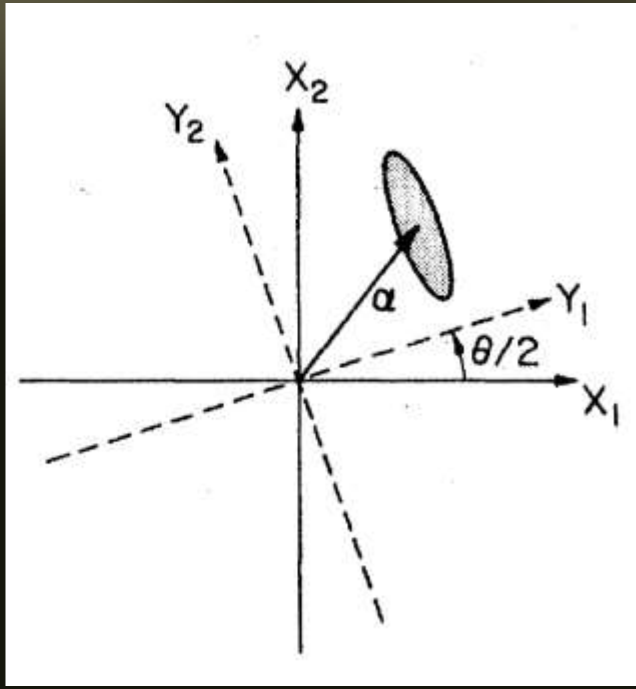
$$(\hat{Y}_1+i\hat{Y}_2) = (\hat{X}_1 + i\hat{X}_2)e^{\frac{-i\theta}{2}}$$

Coherent State

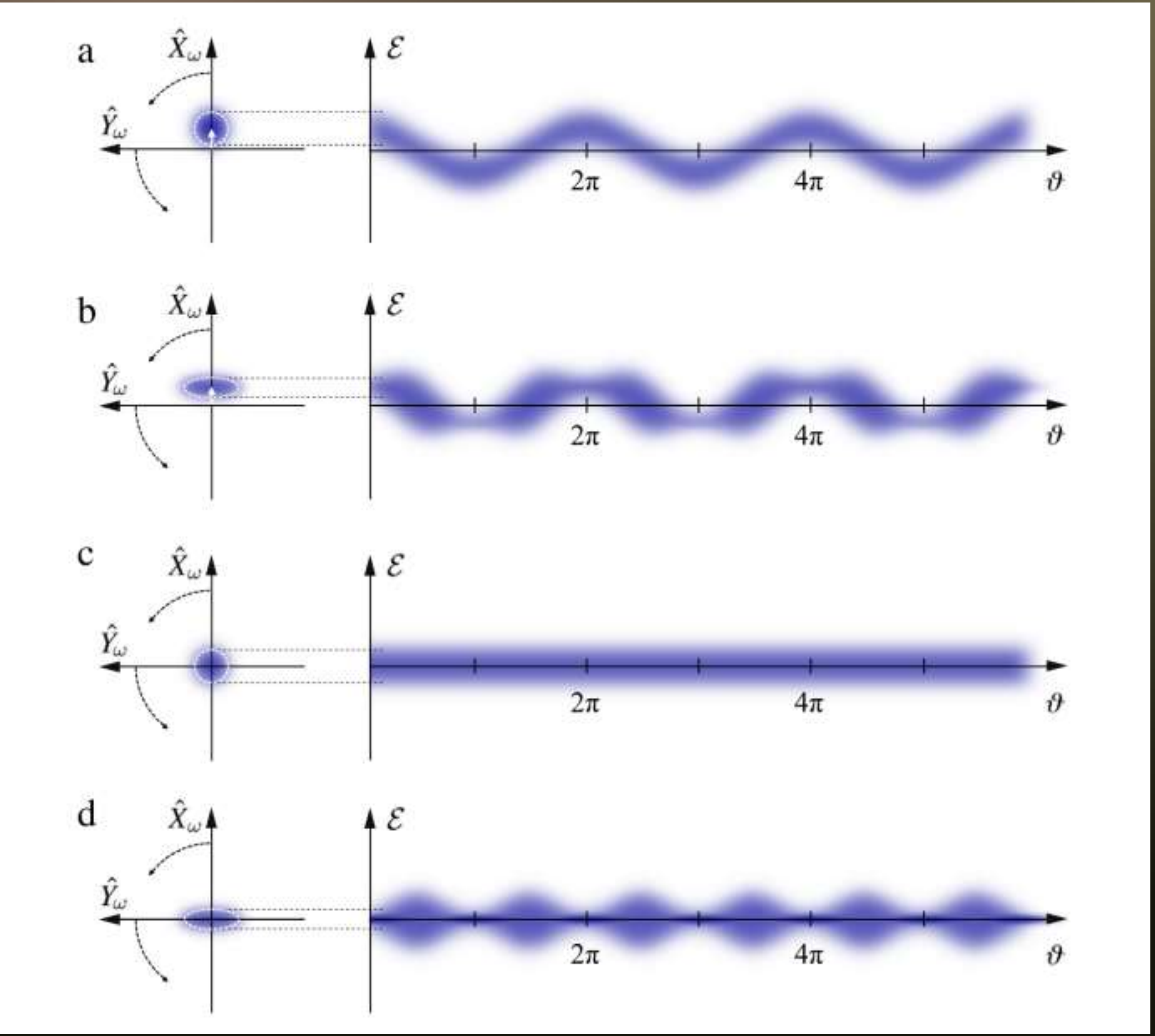


$S(r)$

Squeezed Coherent State

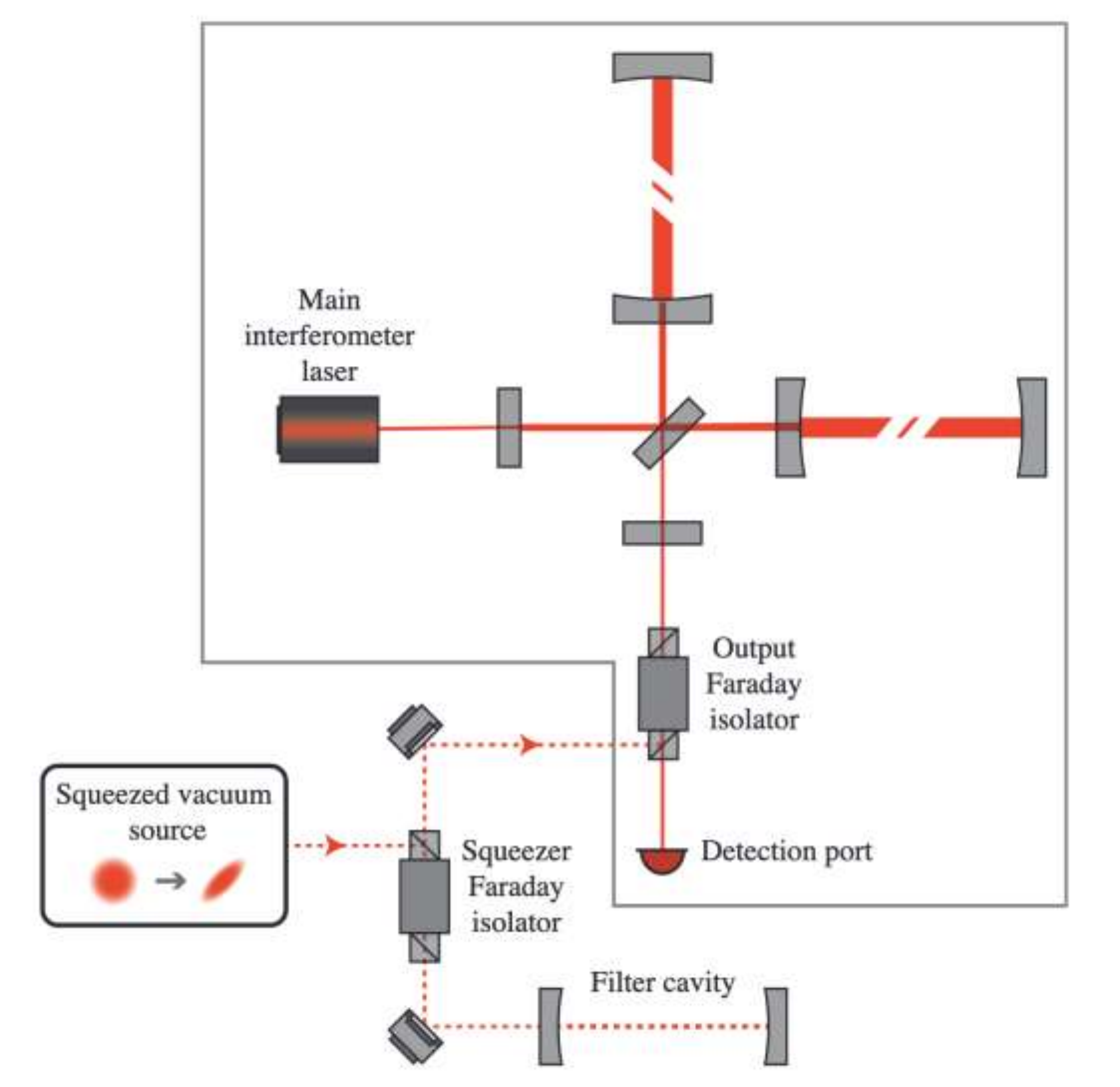


Time Evolution of Squeezed State



Mourou, G. A., Tajima, T., & Bulanov, S. V. (2017).

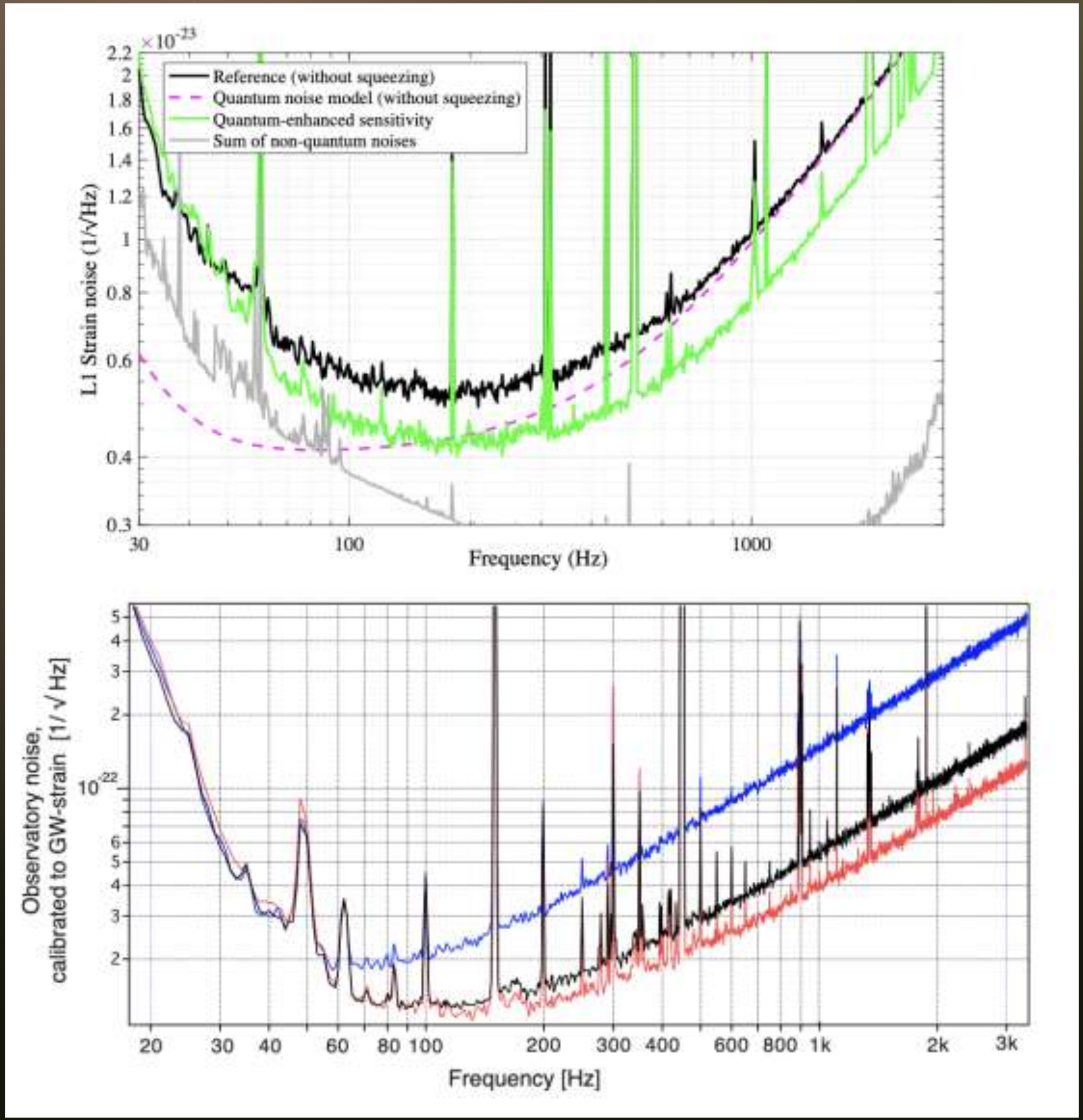
Simplified Squeezed Light Interferometer Implementation



3rd Observing Run Squeezing Data for LIGO and Virgo Detectors

- Top: improved sensitivity of LIGO Livingston Detector by 2.7dB with squeezing

- Bottom: Improved Sensitivity of Virgo by 3.2 dB.
- Blue represents rotation of squeezing by 90 deg



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